Experiment 7: Simple Harmonic Motion

Investigation of the extension and vibrations of a loaded spring

APPARATUS REQUIRED

- spring of unextended length 0.1–0.3 m (force constant 15–20 N m^{-1})
- 9 slotted masses and holder (50 g)
- metre rule
- stopwatch reading to 0.1 s
- 1 stand, 2 clamps and 2 bosses
- 2 wooden blocks for supporting the upper end of the spring
- optical pin and Plasticine
- G-cramp for securing stand base to bench or a 5 kg mass

PRINCIPLES INVOLVED

a Provided that the elastic limit of a spring of natural length l_0 is not exceeded, the extension e is proportional to the load Mg (Fig. 7.1). The constant of proportionality k in the equation:

$$Mg = ke \qquad [7.1]$$

is called the force constant or stiffness of the spring.

k can be simply determined by measuring the increase in length of the spring as Mg is increased. There is only one slight problem: most springs are produced with a slight compression of the turns. Thus for small loads no extension will be registered; it is therefore necessary to place a small load (about 0.5 N) on the spring to remove this compression.

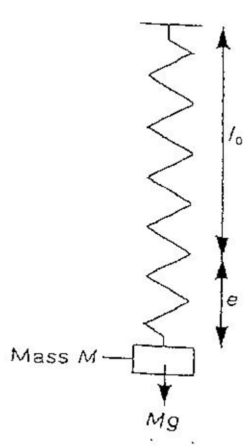

Fig. 7.1. Extension e of a spring of natural length l_0 subject to a force Mg.

b If a mass which produces an extension e is displaced vertically from its equilibrium position by a small amount, the mass executes simple harmonic motion because the restoring force is proportional to the displacement. The period of oscillation T is then given by:

$$T = 2\pi\sqrt{\frac{M}{k}} \qquad [7.2]$$

There are two limitations on the validity of this equation: (i) the spring must not be extended beyond its elastic limit and (ii) the initial further displacement of the spring downwards must not exceed e otherwise the spring will be compressed at the top of its oscillation. Both these factors invalidate equation [7.2].

MEASUREMENT OF THE EXTENSION OF THE SPRING

c Set up the apparatus as shown in Fig. 7.2 (overleaf). Make sure that the spring is firmly held at its upper end. There should be a loop or ring at the lower end of the spring to support the mass holder. Fix the optical pin on the base of the holder with Plasticine.

d Record the reading x_0 indicated by the optical pin on the metre rule when the spring supports the holder alone.

e From this reference point increase M in 50 g steps and record the reading x on the metre rule. Hence calculate the extension $e = (x - x_0)$ for a total mass M (excluding the mass of the holder). Plot a graph of M against e as you load the spring; this will enable you to check that the elastic limit of the spring has not been exceeded.

f From the linear section of your graph, use equation [7.1] to calculate a value for k. Take $g = 9.8$ N kg^{-1} and do not forget to convert M to kilograms.

MEASUREMENT OF THE VIBRATIONS OF A LOADED SPRING

g You may remove the optical pin from the mass holder.

h In measuring the period of oscillation of the loaded spring, you should choose the number of oscillations to be timed so that your timing error of the period is less than 1%. Remember also that the appropriate mass to take is the total mass, including the holder (see equation [7.2]).

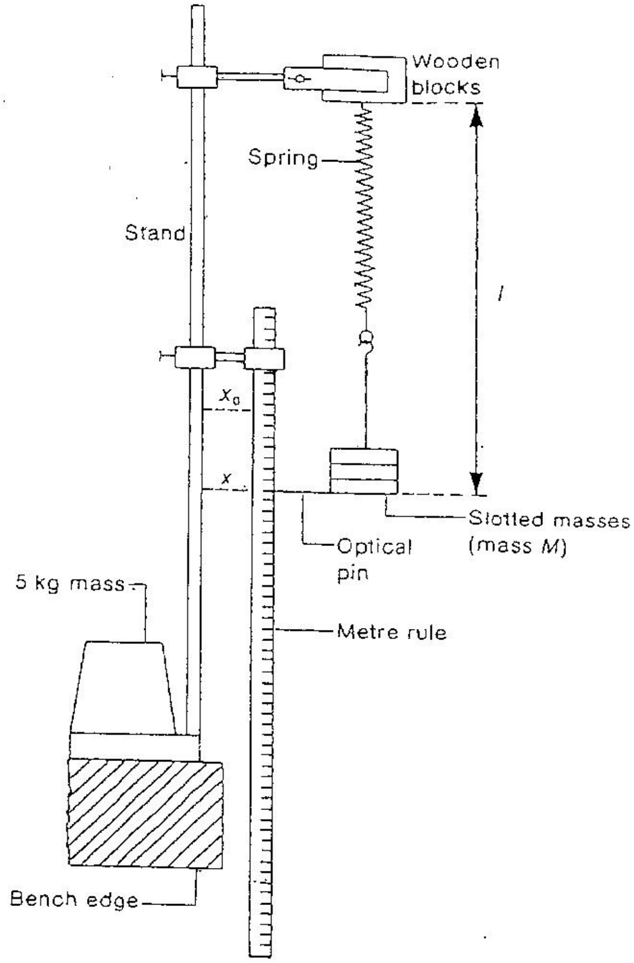

Fig. 7.2 Apparatus for measuring the extension of a spiral spring. The extension for a load Mg is determined from $e = (x - x_0)$.

i Starting with the mass holder alone, determine the period of vertical oscillation T. Draw up a table of M (in kg) and T; leave a further column for data analysis.

j Increase M in 50 g steps and determine T for each value of M. Note that for one particular value of M, you may find it impossible (or difficult) to measure T because the spring alternates quite rapidly between vertical oscillations and horizontal swings (as for a simple pendulum). The periods of these two motions are not the same. If this occurs ignore that reading. You can consider the reasons for this phenomenon in the Further work section.

k From equation [7.2], plot an appropriate graph in order to determine a second value k.

NOTES

l Your account should include a brief description of the experiments, the analysis leading to two values for k. Comment also on any deviations from linearity in the graphs produced in (f) and (k).

m In principle the combined results from the two experiments above can be used to determine a value for g. Substituting for k from equation [7.1] in equation [7.2] $T = 2\pi\sqrt{e/g}$. Thus a graph of T^2 against e should be a straight line of slope $4\pi^2/g$. In practice there is the problem of finding corresponding values of e and M because of the initial compression of most springs.

n In space it is impossible to monitor the 'weight' of an astronaut using normal balances, because the astronaut is in free fall with the space vehicle. A normal spring balance would not register a reading. Thus an ingenious way to measure the astronaut's mass was found: the period of oscillation of the astronaut in a chair supported by springs is measured. Equation [7.2] depends only on the existence of a restoring force proportional to the extension of the spring.