MECHANICAL PROPERTIES OF MATERIALS

Pressure in Solids:

$$Pressure = \frac{force}{area} \qquad \left(P = \frac{F}{A}\right) \qquad \ldots ①$$

Pressure in Liquids:

$$Pressure = height \times density \times gravity \quad (P = h\rho g) \quad \ldots ②$$

Deriving Pressure in a Liquid

Recall that: (i) $volume = area \times height \ (v = Ah)$ ... ③

(ii) $force = mass \times gravity \ (F = mg)$ ... ④

By substituting ④ into ①, we get that the equation:

$$P = \frac{mg}{A} \qquad \ldots ⑤$$

We also know that density can be calculated by using the equation:

$$\rho = \frac{m}{v}$$

And if we substitute ③ into the equation above, we can write that

$$\rho = \frac{m}{Ah} \qquad \ldots ⑥$$

Rearranging ⑥ we can state that

$$A = \frac{m}{\rho h} \qquad \ldots ⑦$$

Substituting ⑦ into ⑤ we get

$$P = mg \div \frac{m}{\rho h}$$

$$P = mg \times \frac{\rho h}{m}$$

$$\therefore P = h\rho g \qquad [\boldsymbol{as\ required}]$$

Phases of Matter

Phases of matter depends on:

- Intermolecular forces
- Motion of molecules due to their internal energy

Note that:

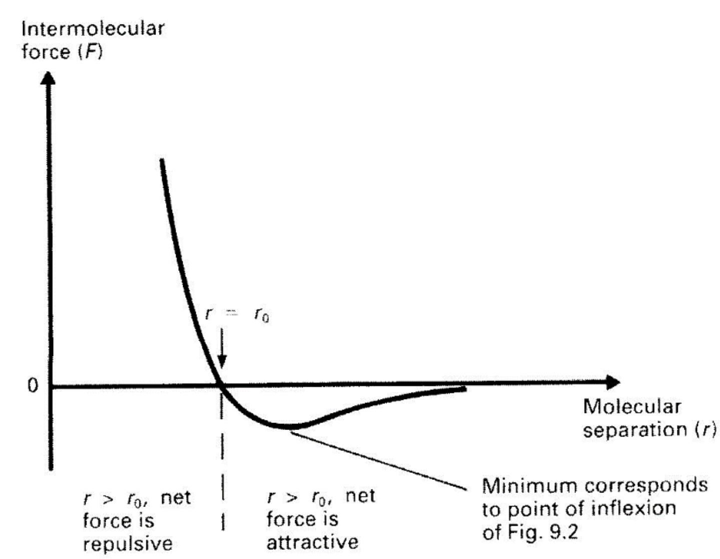
1. When molecules are far apart there are no forces of attraction between them.

2. When molecules are closer together there is a net forces of attraction between molecules.

3. When molecules are too close interpenetration of electrons shells produce a repulsive force.

4. In between ② and ③ there is a separation distance where the net force is zero.

Graph showing forces between a pair of molecules as a function of their separation



Solids

A solid is characterized by a fixed shape because each particle in the solid has a fixed link with its neighbour and so retains a fixed position in the overall pattern (long range order). The molecular or ionic pattern can be of two types:

- Crystalline substances: ordered pattern with periodic reception of the unit cell
- Disordered arrangement of the molecular pattern. (super cooled liquids such as glass)

Liquids

A liquid has a fixed volume but no fixed shape. The molecules of liquids, like those of solids, vibrate. In liquids though, each molecules has a particular set of nearest neighbours for only a short time. This occurs because the molecules of liquids have greater average kinetic energies than those of solids.

Crystalline vs. Non-Crystalline Structures

Crystalline	**Non- Crystalline**
▪ Long range ordering (regularly arrangement of atoms, molecules or ions). Examples: diamond, ice and NaCl ▪ The arrangement that is repeated is called the unit cell. The unit cell is the smallest unit form which the 3D repeating pattern is built (may contain several atoms or molecules) ▪ Structures of crystal depends on the unit cell. The shape depends on (1) the type of bonding in the unit cell and (2) the size and the shape of the bonding particles in the unit cell.	▪ There are two kinds of non-crystalline solids: (1) amorphous and (2) polymeric ▪ Amorphous solids consist of particles possessing short range ordering; packed together in a disorderly arrangement. Hence they do not possess long range ordering (e.g. glass) ▪ Polymers are materials that possess an intermediate state of order between crystalline and amorphous solids. They consist of huge molecules called macromolecules which are made up a long flexible chain of basic groups called monomers (e.g. CH_4). Two types of polymers are: (1) thermosetting polymers which gets harder on heating. (2) thermoplastic polymers which gets softer on heating. (NB. Rubber is a naturally occurring polymer)

Deformation of Solids

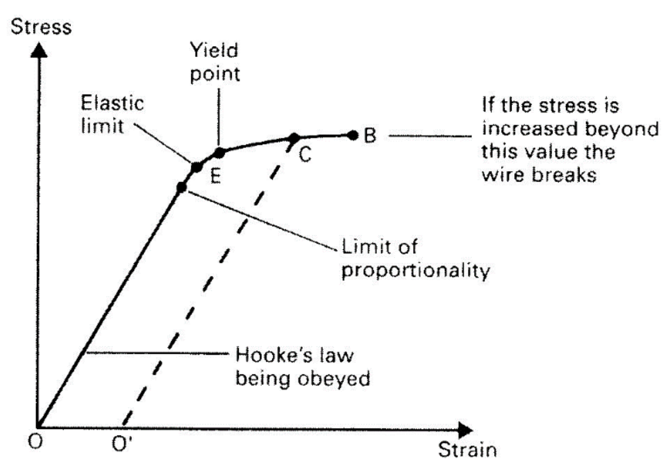
Stress	Force per unit area of cross section (Units: Pascal / Nm^{-2})
Strain	$\frac{change\ in\ length}{original\ length}$ $[no\ units]$
Strength	This is relates to the maximum force which can be applied to a material without breaking it.
Stiffness	This relates to the resistance which a material offers to having its size and /or its shape changed.
Ductility	A ductile material is one which can be permanently stretched.
Brittleness	A brittle material cannot be permanently stretched. It breaks soon after the elastic limit has been reached. Brittle materials are often strong in compression.

When a force acts on a body, it can change the shape and /or size of the body. This will change the relative positions of the molecules within the body. The extension produced in a sample of material depends on:

- the nature of the material
- the force applied
- the cross-sectional area of the sample
- the original length of the sample

The two terms that are used when dealing with the stretching of the material are (1) stress and (2) strain.

If a ductile material such as copper is stretched until it breaks, a graph of stress verses strain for the material resembles that shown below.



Notes:

- ***OE*** is a straight line through the origin indicating that stress α strain. Over this region the material returns to its original length when the stress is removed. The material obeys Hooke's Law in the region ***OE***.

- ***EC*** is a non-linear region where stress is not proportional to strain but the material is till behaving elastically.

- At ***C,*** plastic deformation starts, that is the material does not return to its original length when the force is removed. There is a permanent elongation.

- At ***B*** and beyond, none of the extension is recoverable.

- At ***D (any point beyond B)***, the material breaks.

Young's Modulus (E)

Provided the stress is not so high that the limit of proportionality has been exceeded, the ratio of stress to strain is a constant for a given material. This constant is known as Young's Modulus which is calculated by

$$E = \frac{Stress}{Strain}$$

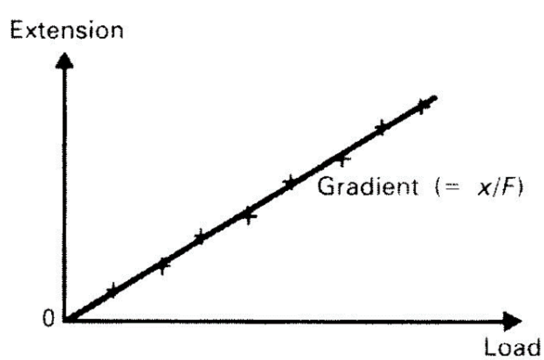
Uints: *Pascal*

Young's Modulus is clearly a measure a material's resistance to changes in length. For example:

E (nautral rubber) = 1.6×10^{6} Nm^{-2}

E (mild) = 2×10^{11} Nm^{-2}

If we determine Young's Modulus by experiment [Muncaster see Page 184], we can obtain a graph of Extension vs. Load.



From the graph we obtain

$$gradient = \frac{x}{F}$$

We also know that

$$E = \frac{Stress}{Strain}$$

And since $Stress = \frac{F}{A}$ and $Strain = \frac{x}{L}$

We can say that

$$E = \frac{F/A}{x/L}$$

Rearranging we get

$$F = \frac{EAx}{L} \qquad \ldots ①$$

We also know that Fα *x* and therefore

$$F = kx$$

$$k = \frac{F}{x} \qquad \ldots ②$$

Rewriting ①and then substituting in ② we get that

$$\frac{F}{x} = \frac{EA}{L}$$

$$k = \frac{EA}{L} \qquad \ldots ③$$

From the graph above and calculating the gradient, we can note that

$$k = \frac{1}{gradient}$$

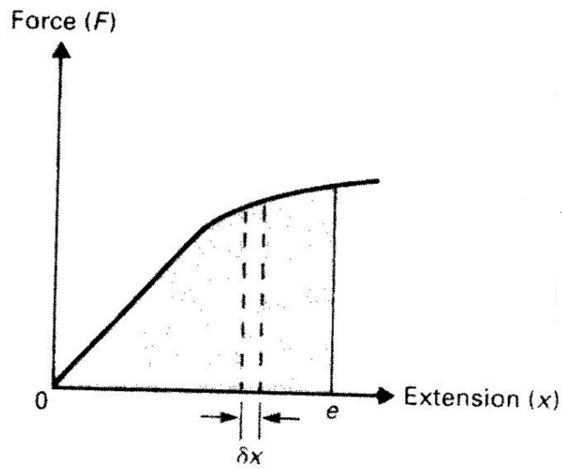
Hooke's Law

This law states that

$$Stress\ \alpha\ \ Strain$$

$$\frac{Stress}{Strain} = [constant]$$

When a force acts on an object, the length of the object changes; as shown in the diagram below.



If the extension (Δl) is small compared to the original length of the object; experiments show that the extension (Δl) is proportional to the force exerted on the object

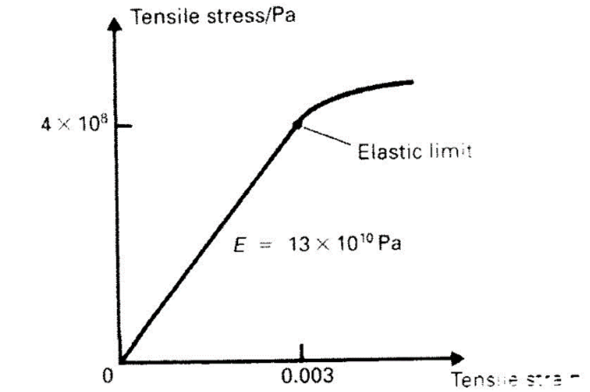
$$F\ \alpha\ \Delta l \qquad (Hooke'sLaw)$$

$$F = k\ \Delta l \qquad (constant\ of\ proportionality)$$

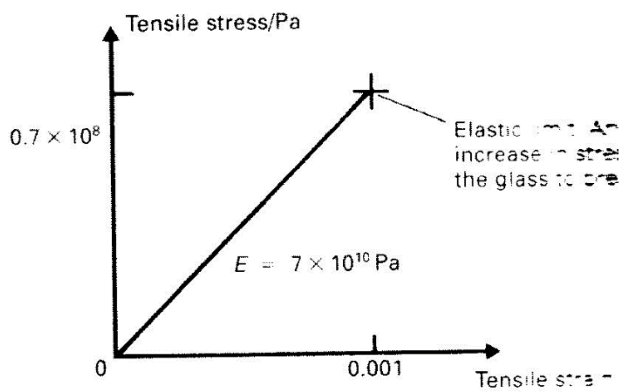
$$k = \frac{F}{\Delta l} \qquad (k\ is\ defined\ as\ force\ per\ unit\ lenght\ extension)$$

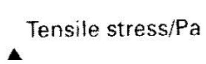
Stress – Strain Curves of Some Typical Samples

Copper

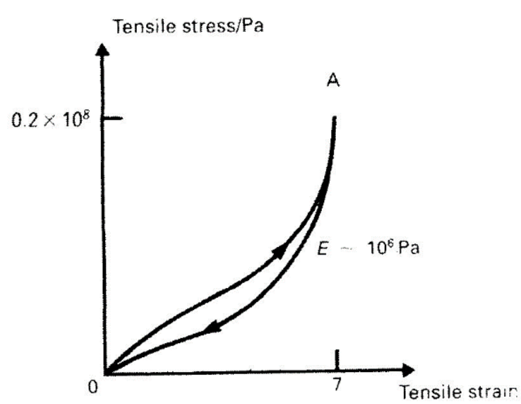


Glass





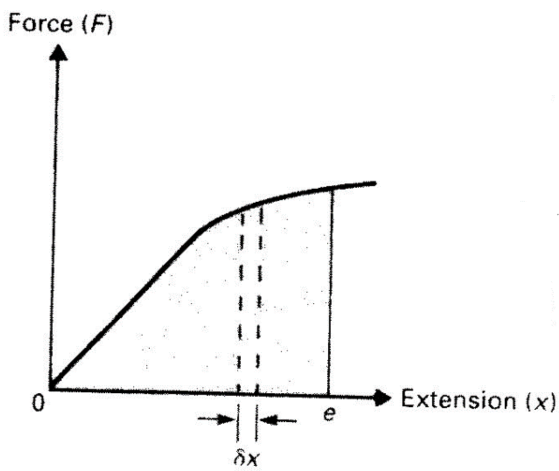
Rubber



Note

Energy Stored in a Stretched Wire

When a wire is stretched without exceeding its elastic limit, the work done on the wire is stored as elastic potential energy in the wire. This energy can be recovered completely provided that the elastic limit is not exceeded. The area under a *Force vs. Extension* Graph gives the work done.



$$area\ under\ the\ graph = work\ done\ in\ producing\ extension$$

$$Area = \frac{1}{2} base \times height$$

$$\frac{1}{2} Fx = \frac{1}{2}(kx)x$$

$$Work\ /Energy\ = \frac{1}{2}\ kx^2$$

A wire of length, L, and a cross sectional area, A, has a volume of AL and therefore the strain energy per unit volume can be calculated by

$$Strain\ energy\ per\ unit\ volume = \frac{\frac{1}{2}\ Fx}{AL}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{x}{L}$$

$$i.e. Strain\ energy\ per\ unit\ volume = \frac{1}{2} \times stress \times strain$$